Density Stratification Effects on Wake Vortex Decay

Dan H. Neuhart,* George C. Greene,† Dale R. Satran,‡ and G. Thomas Holbrook§
NASA Langley Research Center, Hampton, Virginia

Results of an experimental study to determine the effects of density stratification on wake vortex decay are presented. The range of stratification commonly encountered in the atmosphere was simulated in the Vortex Research Facility at NASA Langley Research Center by variations in the vertical temperature gradient. Wake measurements were made for low and high aspect ratio, swept and unswept wings at chord Reynolds numbers of about 2×10^5 to 6×10^5. The results indicate that, in the absence of turbulence, density stratification can determine wake descent distance and lifetime. A comparison of laboratory and flight-test measurements with a vortex decay model suggests a significant Reynolds number effect on wake vortex decay.

Nomenclature

AR = wing aspect ratio $= b^2/S$
b = wing span
b_0 = vortex spacing
c = speed of sound
C_L = lift coefficient = lift$/qS$
g = acceleration due to gravity
H = vertical displacement of vortices; positive up
H^* = nondimensional vertical displacement $= H/b_0$
N = Brunt-Väisälä (B-V) frequency
N^* = nondimensional stratification parameter $= Nb_0/V_0$
p = pressure
P_0 = Brunt-Väisälä period $= 2\pi/N$
q = freesteam dynamic pressure $= \frac{1}{2}\rho U_\infty^2$
R = gas constant for air
R_Γ = vortex Reynolds number $= V_0 b_0/\nu$
s = humidity (air) or salinity (water)
S = wing area
t = time
t_0 = time required for vortex wake to descend one vortex spacing $= b_0/V_0$
t^* = nondimensional displacement time $= tV_0/b_0$
T = absolute temperature
U_∞ = freestream velocity
V_0 = initial vortex descent velocity
z = vertical distance; positive up
β = coefficient of thermal expansion $= -(1/\rho)(\mathrm{d}\rho/\mathrm{d}T)$
γ = ratio of specific heats for air
Γ = circulation
ζ = expansion coefficient for humidity (air) or salinity (water) $= (1/\rho)(\mathrm{d}\rho/\mathrm{d}s)$
ν = kinematic viscosity
ρ = density of a fluid

Subscript
A = adiabatic conditions

Introduction

THE Vortex Research Facility at NASA Langley Research Center has been used for aircraft wake hazard research for more than a decade. Some of the early research in this facility is described in Ref. 1. More recent research has focused on the fundamental physics of vortex decay through detailed measurements of wake structures. During the course of these measurements, it was observed that wake decay characteristics were altered by density stratification in the test area caused by small temperature gradients. A study was initiated to evaluate those effects for the types of models and conditions encountered in the Vortex Research Facility.

The effect of vertical density stratification on the motion and decay of vortex wakes has been the subject of much research in the last 15 years. Reference 2 reviews analytical studies that have yielded conflicting conclusions concerning the effect of stable stratification on wake behavior. Experimental studies, in both laboratories and flight, have uniformly shown that the wake vortex pair decelerates in stable stratification.[3-9] These studies used a wide variation of test facilities and vortex generation and test techniques.

The research reported in this paper was an investigation of wake decay in a quiescent, stratified atmosphere. Wing-generated vortex descent and decay were measured in a towing facility in stratified air. Reynolds number effects were evaluated using an existing theoretical wake decay model.

Experimental Technique

This section is divided into three parts: facility and measurement techniques, stratification scaling parameters, and models. Each of these topics is discussed separately in subsequent sections.

Facility and Measurement Techniques

The Vortex Research Facility (VRF), shown schematically in Fig. 1 and described in Ref. 9, is a converted water towing basin 1800-ft long. An instrumented, automobile-type vehicle is used to accelerate the test model to a speed of approximately 100 ft/s. Model speed is held constant while the vehicle passes through a 500-ft-long test section which contains flow-visualization and recording equipment. A data acquisition system onboard the vehicle obtains model force and moment data and vehicle velocity and acceleration data and optically transmits the data to a 16-bit minicomputer for reduction and postprocessing.

Presented as Paper 85-0050 at the AIAA 23rd Aerospace Sciences Meeting, Reno, NV, Jan. 14-17, 1985; received July 30, 1985; revision received Feb. 20, 1986.

*Project Aerodynamics Engineer, PRC Kentron Inc.
†Research Engineer, Transonic Aerodynamics Division. Member AIAA.
‡Aero-Space Technologist, Analytical Methods Branch, Low Speed Aerodynamics Division. Member AIAA.
§Aero-Space Technologist, Analytical Methods Branch, Low Speed Aerodynamics Division.

The test section is 14-ft high and 17-ft wide. Although it is enclosed within the facility, the environment within the test section is still influenced by the outdoor meteorological conditions. In particular, the temperature near the ceiling tends to track the daily variation in outdoor temperature while the temperature near the floor, which is thick concrete below ground level, is relatively constant. This results in a vertical temperature gradient which varies with the time of day and season. Testing conducted on a typical summer day yielded temperature gradient conditions ranging from near zero in the early morning to a maximum of 0.5–1.0°F/ft (0.9–1.8°C/m) in the afternoon. Tests were conducted at approximately hourly intervals.

Wake motion was determined from flow-visualization measurements using uniform size ($\sim 1\ \mu$m) solid particles for seeding. Two lighting methods were used: a standard floodlight arrangement, and a thin sheet of laser light. The floodlights provided good photographic documentation of the qualitative state of the flow. The laser "sheet of light," perpendicular to the model's line of flight, was created by passing a beam of argon laser light through a cylindrical lens. This lens expanded the circular cross-sectional beam into a 30-deg fan, forming an essentially vertical slit of light extending across the test section. The light reflecting off of the solid particles permitted visual determination of the vortex positions and a qualitative state of flow in the vertical plane without disturbing the test environment. However, attempts to photograph the laser sheet with motion picture or high-speed still cameras were unsuccessful due to the low light levels generated. Therefore, a low-light-level video camera was used to record the laser sheet flow-visualization data.

Scaling Parameters

In order to evaluate the importance of stratification effects in the VRF, scaling relationships were required. A parameter indicative of the degree of stratification in a fluid is the Brunt-Väisälä (B-V) frequency, N. N is the frequency of oscillation of a fluid element displaced from an equilibrium position, i.e., the natural frequency of the vertical buoyancy oscillation. It can be expressed as

$$N = \left[-\frac{g}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}z} - \frac{g^2}{c^2} \right]^{1/2} \tag{1}$$

In general, the density is a function of the pressure, temperature, and other variables, such as the humidity (air) or salinity (water).

If $\rho = \rho(p,T,s)$, then

$$N = \left[g\beta\left(\frac{\mathrm{d}T}{\mathrm{d}z} - \frac{\mathrm{d}T}{\mathrm{d}z}\bigg|_A\right) + g\zeta\left(\frac{\mathrm{d}s}{\mathrm{d}z} - \frac{\mathrm{d}s}{\mathrm{d}z}\bigg|_A\right)\right]^{1/2} \tag{2}$$

For a perfect gas, $\rho = \rho(p,T)$, $\beta = 1/T$, and

$$\frac{\mathrm{d}T}{\mathrm{d}z}\bigg|_A = -\frac{(\gamma-1)}{\gamma}\frac{g}{R} \tag{3}$$

The B-V frequency is then expressed as

$$N = \left[\frac{(\gamma-1)g^2}{\gamma RT} + \frac{g}{T}\frac{\mathrm{d}T}{\mathrm{d}z} \right]^{1/2} \tag{4}$$

For water, which is essentially incompressible, the adiabatic gradients are very small and the B-V frequency becomes

$$N = \left[g\beta\frac{\mathrm{d}T}{\mathrm{d}z} + g\zeta\frac{\mathrm{d}s}{\mathrm{d}z} \right]^{1/2} \tag{5}$$

Thus, in water, stratification can be achieved through temperature gradients, salinity gradients, or both.

In order to have dynamic similarity between full-scale flight conditions and model test conditions, Lissaman et al.[2] pointed out that the ratio of two characteristic times should be the same. One time is proportional to the B-V period, $P_0 = 2\pi/N$, and the other is proportional to the time $t_0 = b_0/V_0$ required for the wake to descend one vortex spacing in an inviscid, unstratified fluid. Although various definitions of the similarity ratio have been used, all are simply related to t_0/P_0. For example, the parameter of Lissaman et al. is equal to $\pi t_0/P_0$, Brown and Kirkman[3] used t_0/P_0, and Hecht et al.[5] define a Froude number as equal to $2P_0/\pi t_0$. The parameter used herein is

$$N^* = 2\pi t_0/P_0 \tag{6}$$

This can be written as

$$N^* = Nb_0/V_0 \tag{7}$$

or in terms of model parameters as

$$N^* = N\frac{4\pi b(b_0/b)^3 \mathit{AR}}{C_L U_\infty} \tag{8}$$

since $b_0/V_0 = 2\pi b_0^2/\Gamma$ and $\Gamma = C_L qS/\rho U_\infty b_0$. In the VRF, the value of V_0 was calculated based on the measured model lift using the equation for line vortices, $V_0 = \Gamma/2\pi b_0$.

For the same N, N^* will be greater for large values of vortex spacing, b_0, coupled with low descent velocities V_0, than for low b_0 and large V_0 (see Eq. 7). These larger N^*s indicate that stratification effects are strong and vortex-pair dynamics are relatively weaker since the time to descend one vortex spacing

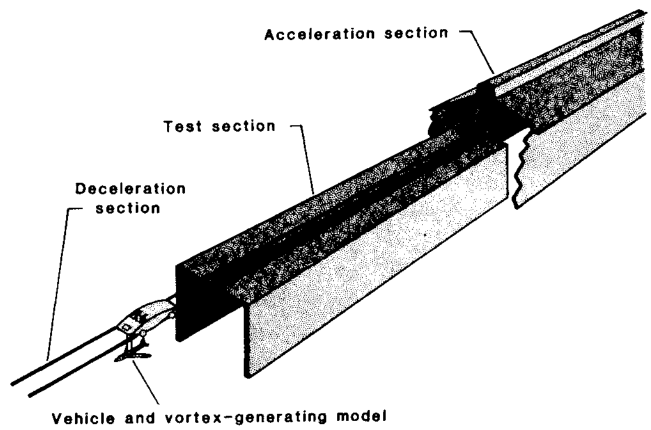

Fig. 1 Schematic diagram of the NASA Langley Vortex Research Facility.

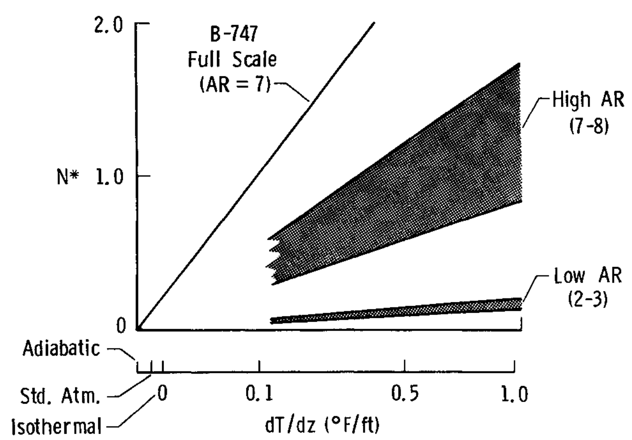

Fig. 2 Relationship between the stratification parameter N^* and the temperature gradient.

is large. Alternately, a lower N^* indicates a dominance of vortex motion over stratification effects.

Figure 2 shows the relationship between the stratification parameter N^* and the temperature gradient. The dT/dz scale is nonlinear in Fig. 2 because the temperature gradient is a nonlinear function of N, as shown in Eq. (4). The top curve shows N^* for a full-scale jumbo jet ($AR = 7$, $C_L = 1.4$). The N^* for this case ranges from 0 for an adiabatic lapse rate to about 1.6 (Ref. 3) in a highly stable atmosphere which might exist above a ground fog. The lapse rates in the atmosphere are typically less than 0.01°F/ft and are too small to be reproduced accurately or controlled in a ground facility. However, by appropriate model selection, the desired N^* may be matched using temperature gradients large enough to be measured accurately. This would imply that the appropriate combination of B-V frequency (N) and model span load (which determines b_0 and V_0) will produce both a model N^* and a wake trajectory similar to full scale. The shaded regions in Fig. 2 show the N^* range for the two types of models tested. Higher aspect ratio models were used to simulate the higher values of N^*, while lower aspect ratio wings were used for small N^*. One scaling parameter not considered in Fig. 2 is the Reynolds number. Recent results using a theoretical model developed by Greene[10] have shown that Reynolds number has a significant effect on wake descent trajectories, as discussed below.

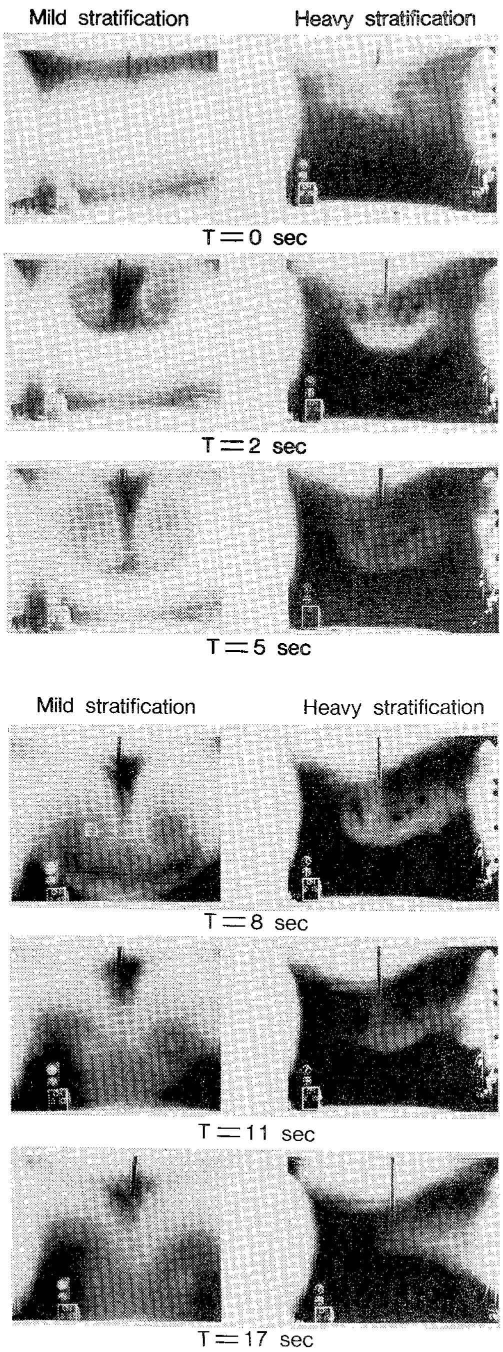

Fig. 3 Stratification effects on the wake of a rectangular wing of aspect ratio 7.

Models

For this investigation, several different planform and aspect ratio models were tested. One model tested was an unswept rectangular wing of aspect ratio 7 with an NACA 0012 airfoil section. This wing had a span of 63.0 in. Two other high aspect ratio wings and one low aspect ratio wing were tested: an aspect ratio 7.8, 21-deg sweptback wing with a span of 32.3 in.; an aspect ratio 8.45, rectangular wing with a span of 32.75 in.; and an aspect ratio 2.79, flat-plate wing having a trapezoidal planform with 60-deg sweptback tips and a span of 27.3 in. The higher aspect ratio models had symmetrical airfoil sections and a simple cone cylinder body around their balance housing. The trapezoidal wing model had a simple semistreamlined body.

Results and Discussion

Stratification effects on wake motion and decay were determined both qualitatively and quantitatively. Qualitatively these effects are shown in a series of photographs in Fig. 3 for the rectangular wing of aspect ratio 7. The left side of the figure shows the wake descent in a temperature gradient of 0.225°F/ft (0.41°C/m). The vortices descended with little apparent decay during the observation period.

The descent of the vortex pair in a more stratified environment (more stable) with a temperature gradient of 1.4°F/ft (2.55°C/m) is shown on the right side of Fig. 3 for the same wing. In this case, the vortices descended a relatively small distance, and shortly thereafter appeared to have completely decayed. It is interesting to note that there was some small-radius or core circulation remaining after descent had stopped (zero net impulse).

The qualitative effect of stratification shown in Fig. 3 is typical of results obtained with all of the tested models and is consistent with all known research including flight tests and laboratory measurements in water and in air.

The quantitative results of wake decay measurements in stratified air in the VRF are shown in Figs. 4–6. The wake trajectories are plotted in nondimensional form in terms of normalized descent distance H^*, normalized time t^*, and the normalized B-V frequency, the stratification parameter N^*. The $H^* = t^*$ line in the figures represents the wake trajectory in an inviscid unstratified fluid, i.e., with the descent velocity remaining constant.

The nondimensional wake descent profiles for the swept wing of aspect ratio 7.8 are shown in Fig. 4. The range of the

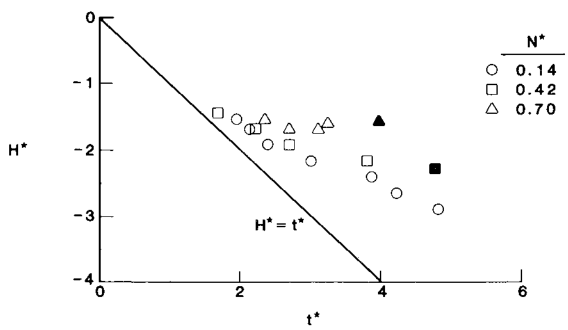

Fig. 4 Stratification effect on wake vortex descent profiles for a 21-deg sweptback wing. $AR = 7.8$.

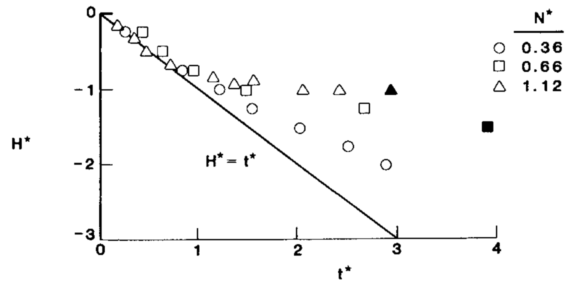

Fig. 5 Stratification effect on wake vortex descent profiles for a rectangular wing, Æ = 8.45.

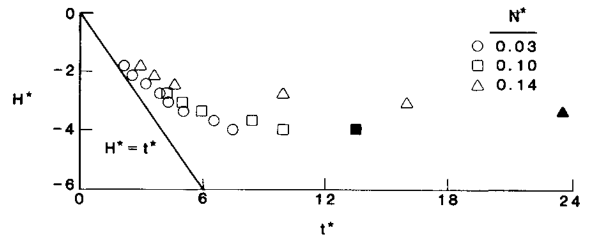

Fig. 6 Stratification effect on wake vortex descent profiles for a wing with a trapezoidal planform, Æ = 2.79.

stratification parameter is 0.14–0.7. The solid symbols in the figure indicate the end of the measured lifetime for each wake. The N^*s for a typical heavy transport aircraft range from 0 (for an adiabatic lapse rate) to about 0.7 for a 0.55°F/100 ft (1°C/100 m) temperature inversion. For a standard atmosphere lapse rate, $N^* \sim 0.2$. Each unit of nondimensional time t^* corresponds to actual times ranging from about 10 s for a single-engine aircraft to about 20 s for a typical heavy transport. In Fig. 4, a lifetime of 5.0 in terms of t^* would imply that a wake from a heavy transport would last for about 100 s (about 4 n.mi. downstream). An example of the wake profile's sensitivity to changes in atmospheric conditions is illustrated by comparing the profiles in Fig. 4 for $N^* = 0.42$ and 0.7. For a typical heavy transport, the $N^* = 0.42$ trajectory represents an isothermal lapse rate of 0°F/100 ft (0°C/100 m). An increase in the lapse rate to 0.55°F/100 ft (1°C/100 m) roughly corresponds to the $N^* = 0.7$ trajectory. This small increase in lapse rate reduces the lifetime of the wake from 100 s to about 80 s which, for a nominal heavy aircraft approach speed, reduces the downstream persistence of the wake by about 0.8 n.mi.—an impressive and desirable reduction in terms of aircraft separation. The wake lifetime for the lowest $N^* = 0.14$ was greater than that for $N^* = 0.42$, but could not be determined because the wake descended out of the region illuminated by the laser sheet of light. Figure 4 shows that increasing levels of density stratification, and thus increasing buoyancy effects on the wake oval, reduced wake lifetime and descent distance.

The wake descent profiles for the rectangular wing of aspect ratio 8.45 are shown in Fig. 5. As with the swept wing, the wake lifetime at the lowest N^* could not be determined. However, the effect due to increasing vertical density stratification was repeated.

The vortex wake descent profiles for the trapezoidal wing (Æ = 2.79) are shown in Fig. 6. The N^* values are small and result in relatively long wake lifetimes. Again, vortex demise for the lowest N^* case was not observed since the wake descended out of the range of the laser sheet. The persistence of the vortices in these wakes indicated a reversal in the trend of reduced lifetime with increasing stratification levels. This remains unexplained. However, the reduction in wake descent distance due to stratification is consistent with the high aspect ratio models.

Shown in Fig. 7 are comparisons of calculated wake trajectories using Greene's vortex wake decay model[10] with low and high Reynolds number test data. The trajectories are presented in nondimensional form to allow comparison between laboratory and full-scale data as implied by the dynamic similarity theory. Figure 7a illustrates the difference between low vortex Reynolds number data taken in the VRF ($R_\Gamma = V_0 b_0 / \nu = 8.4 \times 10^3$) and high Reynolds number data taken in flight by Tombach[7]($R_\Gamma = 3.5 \times 10^5$). Figure 7b shows, at a greater N^*, the difference between VRF data and flight-test data for a full-scale B-747 transport aircraft taken by Burnham et al.[8] ($R_\Gamma = 6.7 \times 10^6$). In both figures, the high Reynolds number wakes descend below the low Reynolds number wakes from the VRF. The theoretical model predicts

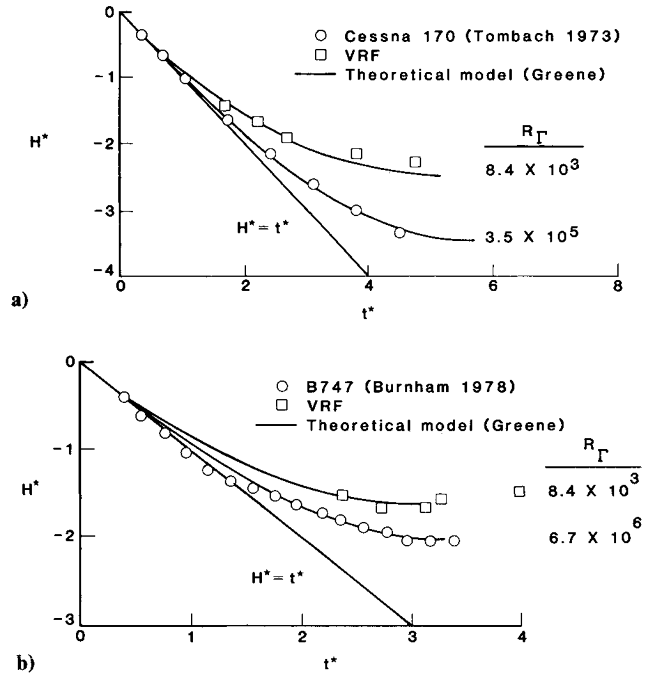

Fig. 7 Reynolds number effect on wake descent: a) $N^* = 0.4$; b) $N^* = 0.7$.

the wake trajectories, and therefore the Reynolds number effects, quite well. An interesting feature of the comparisons is that one would expect a larger difference in maximum descent distance between the two wakes with the largest Reynolds number difference. In this case, however, the wakes with the largest Reynolds number difference were decending in an environment with a larger stratification parameter, N^*. Stratification is known to resist vertical transport, whether it is due to a descending (or ascending) wake or turbulence in general, and as stratification increases, it tends to dominate other factors in the flowfield. Therefore, at increasingly larger N^*s, the Reynolds number effects may diminish.

Interpretation of Results

Taken as a whole, the VRF results showed the same relative trends as a function of stratification as those of Sarpkaya.[6] Sarpkaya presents a rather extensive set of experimental data showing the effect of density stratification on wake motion and decay in a water tank. However, compared to that data, the VRF results generally showed smaller descent distances at comparable values of N^* and t^*. Data taken with high aspect ratio wings were in better agreement than data taken on a wing that had about the same low aspect ratio as those tested in the water tank. In addition, wake decay times for the low aspect ratio wing tested in the VRF were significantly longer.

Reference 6 showed that the wake vortices in the water tank always decayed by linking and/or core bursting following sinusoidal instability for $N^* < 0.75$. The trailing vortices in the

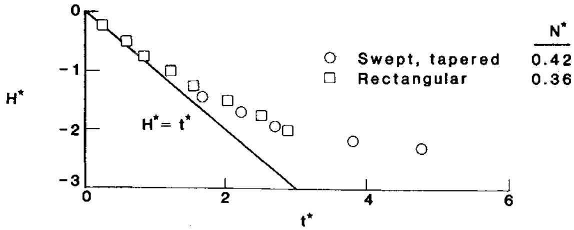

Fig. 8 Descent profiles for a swept wing and a rectangular wing—both with rounded tips.

VRF have not been observed to decay by such means. This may suggest a difference in turbulence level in the two facilities, since turbulence is a forcing function of the sinusoidal instability. In addition to ambient turbulence, model support and transport systems in both fluid mediums are potential sources of turbulence generation. In strongly stratified water ($N^* > 0.75$), however, Sarpkaya observed lower-amplitude sinusoidal instabilities and no vortex linking. This type of flowfield is more closely comparable to those observed in the VRF.

The data of Ref. 6 showed that planform shape was not significant in determining ascent profiles (the models lifted downward and, therefore, their wakes ascended). But wing-tip shape or vortex core size had a significant effect. Sharp-edged tips were found to produce larger vortex cores and less vortex ascent, and rounded tips produced tighter cores and larger vortex rise. The two high aspect ratio models whose VRF descent profiles are shown in Figs. 4 and 5 had rounded tips but showed less, rather than more, descent distance at corresponding times than Sarpkaya's sharp-edged wings. However, in the present investigation, the models with rounded tips did show similar descent profiles at similar N^* values (see Fig. 8). It is believed that consistent results were obtained from the models tested in the VRF.

It appears that the primary source of the differences between the VRF results and the water-tank data was the method of determining V_0 used for nondimensionalizing time t and the B-V frequency N. Sarpkaya indirectly measured V_0 using initial vortex trajectory data. The V_0 values for the VRF data were calculated as a function of measured model lift coefficients for the inviscid case. It is believed that this method is more consistent for the VRF since initial trajectory data were not usually available due to the limited range of the laser sheet of light, and because the vortex cores shed at the wing trailing edge do not have the complete bound circulation wrapped into them. Therefore, their descent speed is not the wake oval descent speed. Corrections for these differences would probably bring the two data sets into closer agreement, except for the cases in the VRF where very persistent vortices were observed. In these cases, t^* values as high as 13–15 were obtained, which is over twice the lifetime observed by Sarpkaya. However, in flight tests under stable, low-turbulence conditions, wake lifetimes of about $t^* = 13$ have been observed.

Conclusions

Recent tests in the NASA Langley Research Center Vortex Research Facility (VRF) have confirmed that density stratification (produced by vertical temperature gradients) has a large effect on the decay of the wake vortex system behind an aircraft. Significant reduction of the vortex wake descent distances and lifetimes was observed in stably stratified air. Comparisons made with existing extensive water-tank data verified the trends observed for vortex descent profiles in the VRF. The effect of Reynolds number on wake descent was identified by comparing laboratory and flight-test results and by using a theoretical model. These results indicate that as the stratification increases and dominates the flowfield, the effect of Reynolds number may become less important.

References

[1]Patterson, J.C., "Lift Induced Wing Tip Vortex Attenuation," AIAA Paper 74-38, Jan. 1974.

[2]Lissaman, P.B.S., Crow, S.C., MacCready, P.B. Jr., Tombach, I.H., and Bate, E.R. Jr., "Aircraft Vortex Wake Descent and Decay Under Real Atmospheric Effects," FAA-RD-3-120, Oct. 1973.

[3]Brown, C.E. and Kirkman, K., "Simulation of Wake Vortices Descending in A Stably Stratified Atmosphere," FAA-RD-74-116, July 1974.

[4]Tomassian, J.D., "The Motion of a Vortex Pair in a Stratified Medium," Ph.D. Dissertation, University of California-Los Angeles, June 1979.

[5]Hecht, A.M., Hirsh, J., and Bilanin, A.J., "Turbulent Trailing Vortices in Stratified Fluids," AIAA Paper 80-0009, Jan. 1980.

[6]Sarpkaya, T., "Trailing Vortices in Homogeneous and Density-Stratified Media," *Journal of Fluid Mechanics,* Vol. 136, Nov. 1983, pp. 85-109.

[7]Tombach, I., "Observations of Atmospheric Effects on the Transport and Decay of Trailing Vortex Wakes," AIAA Paper 73-110, Jan. 1973.

[8]Burnham, D.C., Hallock, J.N., Tombach, I.H., Brashears, M.R., and Barber, M.R., "Ground-Based Measurements of a B-747 Aircraft in Various Configurations," FAA-RD-78-146, Dec. 1978.

[9]Satran, D.R., Neuhart, D., Holbrook, G.T., and Greene, G.C., "Vortex Research Facility Improvements and Preliminary Density Stratification Effects on Vortex Wakes," AIAA Paper 85-0050, Jan. 1985.

[10]Greene, G.C., "An Approximate Model of Vortex Decay in the Atmosphere," AIAA Paper 85-1835, Aug. 1985.